Grand Unified Theory based on the $SU(6)$ symmetry

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Abstract

We present a complete set of generators for the rank 5 special unitary group, $SU(6)$, to unify strong, weak and electromagnetic interactions. The unification is realized through the breaking pattern of $SU(6) \to SU(3)_C \otimes SU(3)_H \otimes U(1)_C$ followed by $SU(3)_H \to SU(2)_L \otimes U(1)_B$. All known elementary particles and its quantum numbers are well accomodated in its $\{6\}$ and $\{15\}$ multiplets. These multiplets require a new neutral fermion which should be assigned as the heavy Majorana neutrino to realize the seesaw mechanism naturally in the minimal scenario of this model.

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1 Introduction

Nowadays, all phenomenons in the high energy physics have been explained within the standard model (SM) which is a gauge theory based on the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry [1]. This set of symmetry represents strong, weak and electromagnetic interactions in a universal framework. In contrast to the weak and electromagnetic interactions which have been successfully unified in the electroweak theory based on $SU(2)_L \otimes U(1)_Y$ symmetry, the strong interaction with $SU(3)_C$ symmetry remains independent from the others.

So far, the electroweak theory is in impressive agreement with the most of experimental observables [2]. However, there are recently several experimental results which disagree with the SM's predictions, as the oscilation in the neutrino sector [3] and the discrepancy in the NuTeV measurement [4]. There are also undergoing or forthcoming experiments to measure the double β decay [5], to search the Higgs particle(s) needed to break the symmetry [6], to relate the high energy phenomenon with the cosmology one and so on. All of them have been expected to be able to distuingish some physics beyond the SM. As mentioned above, the SM is lacking of explaining the unification of three gauge couplings at a particular scale, especially under an assumption that our nature should be explained by a single unified theory, the so called grand unified theory (GUT).

In order to realize GUT at some scale, most of works in the last decades have dealt with gauge theory inspired by the successfull electroweak theory. Those theories assumed the gauge invariance under particular symmetries larger than the SM's one, but contain $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as a part of its subgroups at electroweak scale. Just to mention, the most famous models in this category are $SU(5)$ [7] and $SO(10)$ [8]. Thereafter there are a lot of variants of these models including the supersymmetrized ones. There were also few works on $SU(6)$ GUT with and without supersymmetry [10, 11].

In this paper, we follow the same approach to extend the SM by introducing larger symmetry based on $SU(6)$ group. Since this group has rank 5, same as $SO(10)$, it will provide a new alternative to realize gauge unification beside $SO(10)$ GUT. Higher rank than $SU(5)$ implies that the $SU(6)$ GUT has several scales before breaking down to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. This is an important feature to avoid too fast proton decay [9].

Before going further, we would like to point out the differences between the present paper and the previous works in $SU(6)$ GUT [10, 11]. The pioneering work [10] has dealt with the $SU(6)$ GUT which breaks down to the SM through the breaking pattern in the first stage as $SU(6) \rightarrow SU(5) \otimes U(1)$ by introducing an extra $U(1)$ gauge symmetry. This was highly motivated by a desire to incorporate the $SU(5)$ GUT which was the most potential candidate for GUT that time. Some subsequent works [11] then extended the model to its supersymmetric versions. In contrast, in the present work we follow different breaking pattern, *i.e.* $SU(6) \rightarrow SU(3) \otimes SU(3) \otimes U(1)$. Moreover, we also begin from constructing the $SU(6)$ group itself, that is determining explicitly the generators which could realize the assumed symmetry breaking. Using the extended Gell-Mann Okubo relation which is derived from the Cartan sub-algebra in the group, we assign the fermions in the appropriate multiplets. Actually, we propose completely different fermion multiplets in the {6}

and $\{\overline{15}\}$ representations. Therefore, the model is entirely different from the previous $SU(6)$ GUT models.

The paper is organized as follow. First, we discuss appropriate symmetry breakings in $SU(6)$ GUT in Sec. 2. Based on these breaking patterns, the basic of $SU(6)$ group and its generators are then given in Sec. 3. Before presenting the extended Gell-Mann Okubo relation, we perform a detail study of the quantum numbers contained in the model in Sec. 4. Using this extended Gell-Mann Okubo relation, we propose a new particle assignment in the $SU(6)$ multiplets, *i.e.* $\{6\}$ and $\{\overline{15}\}$ in Sec. 5. We show that in this configuration at least a new neutral fermion is naturally required. This could then be interpreted as the heavy Majorana neutrino that is crucial in the so-called seesaw mechanism needed to explain very light observed neutrinos.

2   Pattern of symmetry breaking

First of all, determining the pattern of symmetry breaking in a GUT model is a crucial step. In the case of $SU(6)$ group, concerning only the sub-matrices of its generators, intuitively there are several possibilities to break the symmetry, for example

$$SU(6) \rightarrow \begin{cases} SU(5) \otimes U(1) \\ SU(2) \otimes SU(2) \otimes SU(2) \otimes U(1) \otimes U(1) \\ SU(3) \otimes SU(3) \otimes U(1) \end{cases} . \tag{1}$$

The first choice is clearly similar to the known $SU(5)$ GUT where it is followed by the breaking pattern of $SU(5) \rightarrow SU(3) \otimes SU(2) \otimes U(1)$ to obtain the SM. This breaking pattern has been introduced by [10], however this is not much preferred due to too fast proton decay. On the other hand, the second example can be excluded since it is not able to accomodate the SM. Then the last one is the only pattern we should choice and it has actually not been studied so far.

At this present stage, we can straightforward put the first $SU(3)$ as $SU(3)_{\mathrm{C}}$ representing the strong interaction, while the second one should break further to $SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{Y}}$ to reproduce the electroweak theory. So, there are two stages to break $SU(6)$ down to the electroweak scale,

$$\begin{aligned} SU(6) &\rightarrow SU(3)_{\mathrm{C}} \otimes SU(3)_{\mathrm{H}} \otimes U(1)_{\mathrm{C}} \\ &\rightarrow SU(3)_{\mathrm{C}} \otimes SU(2)_{\mathrm{L}} \otimes U(1)_{\mathrm{B}} \otimes U(1)_{\mathrm{C}} , \end{aligned} \tag{2}$$

where H denotes a new quantum number which we later on call as hyper-isospin. The combination of quantum numbers induced by $U(1)_{\mathrm{B}}$ and $U(1)_{\mathrm{C}}$ will reproduce the familiar hypercharge associated with $U(1)_{\mathrm{Y}}$ in the electroweak theory. These points will be clarified in detail in Sec. 4.

Next, we should consider the fundamental representations and the minimal multiplets to accomodate the particle contents. The fundamental representations of $SU(6)$ group is represented as $\{6\}$ and its anti-symmetric $\{\overline{6}\}$. A tensor product of two fundamental representations gives, $\{6\} \otimes \{6\} = \{21\} \oplus \{\overline{15}\}$. Following the general requirement for the anomaly free combination of representations of fermions in any particular $SU(N)$ group [12], one should choose the combination of $2\{6\} \oplus \{\overline{15}\}$ in the case of $SU(6)$. The second $\{6\}-$dimensional representation comes up from

the decomposition of {21} in the above tensor product. Therefore we can conclude here that the fermions must be assigned in these multiplets, namely sextet ({6}) and decapentuplet ({15}). The particle contents in each multiplet will be given in Sec. 5 after deriving the extended Gell-Mann Okubo relation in Sec. 4.

3 $SU(6)$ group

In this section, we construct the generators for $SU(6)$ group. In general, the generators for $SU(N)$ group can be determined using the existing generators of $SU(N-1)$ group and expanding its $(N-1) \times (N-1)$ matrices to be $N \times N$ matrices [13]. Then there are three considerable types of matrices which could form an $SU(N)$ group,

$$\lambda_i = \begin{cases} \begin{pmatrix} \begin{pmatrix} & & & \\ & \tilde{\lambda}_i & & \\ & & & \end{pmatrix} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \begin{matrix} 0 & \cdots & 0 \end{matrix} & 0 \end{pmatrix} & , \text{ for } i = 1, 2, \cdots, (N-1)^2 - 1 \\[3em] \begin{pmatrix} (0)_{(N-1)\times(N-1)} & \begin{matrix} 0 \\ \vdots \\ a_{jN} \\ \vdots \\ 0 \end{matrix} \\ \begin{matrix} 0 \cdots a_{Nj} \cdots 0 \end{matrix} & 0 \end{pmatrix} & , \text{ for } (N-1)^2 - 1 < i < N^2 - 1 \\[3em] \lambda_{N^2-1} & , \text{ for } i = N^2 - 1 \end{cases} \qquad (3)$$

where $\tilde{\lambda}_i$ is the $i-$th generator of $SU(N-1)$ group and $a_{jN} = a_{Nj}^* = 1$ or $-i$ with $j = 1, 2, \cdots, N-1$. This confirms that the total number of generators in an $SU(N)$ group equals to $[(N-1)^2 - 1] + [2 \times (N-1)] + 1 = N^2 - 1$. Note that special (physical) consideration must be taken to determine the last generator, *i.e.* λ_{N^2-1} beside the basic mathematical requirement $\text{tr}(\lambda_i \lambda_j) = 2\delta_{ij}$. Of course one should remark that the order of numbering the generators can be changed for the sake of convenience due to some physical considerations as discussed soon. Now we are ready to move forward to the case of $SU(6)$ group.

Throughout the paper we use the notation $\overline{\lambda}_i$ to indicate the generators of $SU(3)$ (Gell-Mann matrices [15]), $\tilde{\lambda}_i$ for $SU(5)$, λ_i for the $SU(6)$ and $\sigma_{1,2,3}$ for the Pauli matrices. We start from the well-known generators of $SU(5)$ [14]. It is considerable to bring the $\tilde{\lambda}_{1,\cdots,20}$ as they are and extend them to be $\lambda_{1,\cdots,20}$ by adding the $6-$th rows and columns with null elements. This implies that the color quantum number is preserved as the conventional quantum chromodynamics (QCD), *i.e.* the upper left 3×3 block still represents the $SU(3)_\text{C}$ symmetry. Since the last generator should form the Cartan sub-algebra which determines the (physically meaningfull) eigen values, that is having non-zero diagonal elements, we eliminated the $\tilde{\lambda}_{24}$. Instead of that we put $\lambda_{21,\cdots,26}$ as the type of extended σ_1 and σ_2 matrices filling in the upper-right and lower-left 3×3 blocks.

Further, $\tilde{\lambda}_{21,22,23}$ are kept and extended to be $\lambda_{27,28,29}$ to represent the $SU(3)_{\mathrm{H}}$ group after the first step of symmetry breaking in Eq. (2). The extended σ_1 and σ_2 types with its non-zero elements filling the last rows and columns in the lower-right 3×3 block form $\lambda_{30,31,32,33}$.

Since $SU(6)$ is a rank 5 group, it should have five generators form its Cartan sub-algebra. In a more technical term, there must be five generators with non-zero diagonal elements. Since we already have three of them ($\lambda_{3,8,29}$), therefore we should define the remaining two diagonal generators. From the fact that $\lambda_{27,\cdots,33}$ have the same form as the extended $\overline{\lambda}_{1,\cdots,7}$ of $SU(3)$, it is then appropriate to choose λ_{34} as the extended form of $\overline{\lambda}_8$.

As mentioned earlier, we must take physical considerations to determine the remaining λ_{35}. Concerning the first step of symmetry breaking in Eq. (2), λ_{35} should reflect the quantum number of $U(1)_{\mathrm{C}}$ and be independent from both $SU(3)_{\mathrm{C}}$ and $SU(3)_{\mathrm{H}}$. It yields that,

$$\lambda_{35} = \frac{1}{\sqrt{3}} \left(\begin{array}{c|c} (-1)_{3\times3} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (1)_{3\times3} \end{array} \right) . \tag{4}$$

Finally, the generators for $SU(6)$ group can be defined in a common way using these matrices as follows,

$$F_i \equiv \frac{1}{2} \lambda_i \quad (i : 1, \cdots, 35) , \tag{5}$$

which satisfies the relation $[F_i, F_j] = i f_{ijk} F_k$ with f_{ijk} is the structure constant respectively. Complete expressions for all matrices are given in the Appendix.

Before going on to the next section, we would like to make several remarks here,

- The Gell-Mann like matrices $\lambda_{27,\cdots,34}$ with non-zero elements in the lower-right 3×3 block represents the $SU(3)_{\mathrm{H}}$ at the first symmetry breaking. This generates a new quantum number namely hyper-isospin.

- Since $SU(6)$ contains $SU(5)$ as its sub-group, $\tilde{\lambda}_{24}$ should be able to be contained in a 6×6 matrix which is the linear combination of λ_{34} and λ_{35}, *i.e.* ,

$$c_{34}\lambda_{34} + c_{35}\lambda_{35} = \frac{2}{\sqrt{15}} \left(\begin{array}{ccccc|c} 1 & & & & & 0 \\ & 1 & & & & \\ & & 1 & & & \vdots \\ & & & -\frac{3}{2} & & \\ & & & & -\frac{3}{2} & 0 \\ \hline 0 & \cdots & & & 0 & 0 \end{array} \right) = \left(\begin{array}{c|c} \tilde{\lambda}_{24} & 0 \\ \hline 0 & 0 \end{array} \right) , \tag{6}$$

where the multiplication factors are choosed to be $c_{34} = -1/\sqrt{5}$ and $c_{35} = -2/\sqrt{5}$.

- λ_{35} represents the hypercharges exist in the strong and weak forces with opposite signs. This reflects the property of its short and long range interactions. We label this kind of hypercharge as $C-$hypercharge.

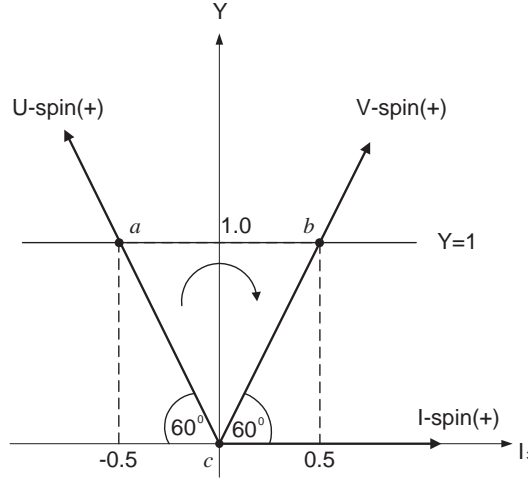- On the other hand, the hypercharge induced by λ_{34} exists only in the weak sector. We label it as $B-$hypercharge.

Figure 1: $I-$, $U-$ and $V-$ spin on the $I_3 - Y$ plane for a, b and c in the $SU(3)_\text{H}$ triplet.

4 Quantum number

Since the $SU(3)_\text{C}$ symmetry is kept till the low energy scale, in the sense of quantum number there is no new physical consequence on it. Then, let us focus on the generators form $SU(3)_\text{H}$ relevant for the electroweak interaction. We should reconsider the Gell-Mann Okubo relation which has been well established within the SM. This relation constitutes that the isospin and hypercharge are the constituents of charge, *i.e.* $Q = I_3 + \frac{1}{2}Y$. In the present framework we have several new hypercharges as mentioned in the preceeding section. This motivates us to consider the extended Gell-Mann Okubo relation.

The $B-$hypercharge induced by λ_{34} has non-identical hypercharges, *i.e.* (1 1 $-$ 2), in contrast with the identical $C-$hypercharge, (1 1 1). If the total hypercharge Y is defined as,

$$Y \equiv Y_B + Y_C , \tag{7}$$

we obtain non-identical hypercharge and isospin configurations for the doublets which are possibly built from $SU(3)_\text{H}$ triplet. This strange behaviour can be explained by introducing the $U-$, $V-$ spins beside the conventional $I-$spin. If a, b, c denote the first, second and third elements in the $SU(3)_\text{H}$ triplet, there are three combinations of doublet respectively,

$$I - \text{spin} : \begin{pmatrix} b \\ a \end{pmatrix} , \quad U - \text{spin} : \begin{pmatrix} a \\ c \end{pmatrix} , \quad V - \text{spin} : \begin{pmatrix} c \\ b \end{pmatrix} . \tag{8}$$

These combinations can be illustrated on the $I_3 - Y$ plane as shown in Fig. 1. Note that the conventional isospin I_3 is determined by λ_{29}.

From these results, we have found that the third component of hyper-isospin I_H is related with isospin and hypercharge as follows,

$$I_{H_3} = I_3 + (\Delta I_3 \cdot \Delta Y) , \tag{9}$$

where the delta means the difference between the upper and lower elements in each doublet (see Eq. (8)). Secondly, the charge for each element can be derived using

this hyper-isospin and the total hypercharge,

$$Q = I_{H_3} + \frac{1}{2}Y \tag{10}$$

$$= I_3 + [\Delta I_3 \cdot (\Delta Y_B + \Delta Y_C)] + \frac{1}{2}(Y_B + Y_C) \ ,$$

using Eqs. (7) and (9). This is the extended Gell-Mann Okubo relation in the framework of $SU(6)$ GUT under consideration.

5 Particle assignment

With the extended Gell-Mann Okubo relation at hand, we are now ready to go on assigning the particle contents appropriately. As mentioned earlier we must fill all fermions in the combination of the $\{6\}-$ and $\{\overline{15}\}-$plets.

Taking into account the quantum numbers (charge, isospin and hypercharge) defined above, we should take,

$$(\psi^6)^i_R = \begin{pmatrix} d^i_r \\ d^i_b \\ d^i_g \\ (\ell^i)^+ \\ -(\nu^i_\ell)^C \\ N_{\ell i} \end{pmatrix}_R , \tag{11}$$

for the sextet, while the $\{\overline{15}\}-$plet should consist of,

$$(\psi^{15})^{ij}_L = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & (u^i_g)^C & -(u^i_b)^C & -u^i_r & -d^i_r & -d^j_r \\ -(u^i_g)^C & 0 & (u^i_r)^C & -u^i_b & -d^i_b & -d^j_b \\ (u^i_b)^C & -(u^i_r)^C & 0 & -u^i_g & -d^i_g & -d^j_g \\ u^i_r & u^i_b & u^i_g & 0 & (\ell^j)^+ & -(\ell^i)^+ \\ d^i_r & d^i_b & d^i_g & -(\ell^j)^+ & 0 & (N_{\ell i})^C \\ d^j_r & d^j_b & d^j_g & (\ell^i)^+ & -(N_{\ell i})^C & 0 \end{pmatrix}_L , \tag{12}$$

where $u^i : u, c, t$; $d^i : d, s, b$; $\ell^i : e, \mu, \tau$; $N_{\ell i} : N_e, N_\mu, N_\tau$ and r, g, b denote the colors respectively. N_ℓ's are newly introduced fermions with neutral charges. Note that i, j denote the generation and its combination goes cyclic, *i.e.* $(i, j) : (1, 2) \to (2, 3) \to (3, 1)$. L and R are the projection operators, $L \equiv \frac{1}{2}(1 - \gamma_5)$ and $R \equiv \frac{1}{2}(1 + \gamma_5)$.

We should make few remarks here. First, we assign the identical fermions for two sextets required to avoid the anomaly. Secondly, it is clear that this model on its own implies the existence of a new neutral fermion, N_ℓ, to complete its multiplets. This exotic fermion then could be interpreted as the heavy Majorana neutrino to enable the seesaw mechanism naturally. Lastly, this is clearly the minimal particle assignment in the present model, *i.e.* the minimal $SU(6)$ GUT. One could also take other possibilities by introducing more exotic fermions as done in [10].

According to Eqs. (11) and (12), $(\psi^6)^i_r$ can be written explicitly for each gener-

ation as,

$$
(\psi^6)^1_R = \begin{pmatrix} d_r \\ d_b \\ d_g \\ e^+ \\ -(\nu_e)^C \\ N_e \end{pmatrix}_R \quad , \quad
(\psi^6)^2_R = \begin{pmatrix} s_r \\ s_b \\ s_g \\ \mu^+ \\ -(\nu_\mu)^C \\ N_\mu \end{pmatrix}_R \quad , \quad
(\psi^6)^3_R = \begin{pmatrix} b_r \\ b_b \\ b_g \\ \tau^+ \\ -(\nu_\tau)^C \\ N_\tau \end{pmatrix}_R \quad ,
$$
(13)

while the contents of $(\psi^{15})^{ij}_L$ are,

$$
(\psi^{15})^{12}_L = \frac{1}{\sqrt{2}}
\begin{pmatrix}
0 & (u_g)^C & -(u_b)^C & -u_r & -d_r & -s_r \\
-(u_g)^C & 0 & (u_r)^C & -u_b & -d_b & -s_b \\
(u_b)^C & -(u_r)^C & 0 & -u_g & -d_g & -s_g \\
u_r & u_b & u_g & 0 & \mu^+ & -e^+ \\
d_r & d_b & d_g & -\mu^+ & 0 & (N_e)^C \\
s_r & s_b & s_g & e^+ & -(N_e)^C & 0
\end{pmatrix}_L \quad ,
$$

$$
(\psi^{15})^{23}_L = \frac{1}{\sqrt{2}}
\begin{pmatrix}
0 & (c_g)^C & -(c_b)^C & -c_r & -s_r & -b_r \\
-(c_g)^C & 0 & (c_r)^C & -c_b & -s_b & -b_b \\
(c_b)^C & -(c_r)^C & 0 & -c_g & -s_g & -b_g \\
c_r & c_b & c_g & 0 & \tau^+ & -\mu^+ \\
s_r & s_b & s_g & -\tau^+ & 0 & (N_\mu)^C \\
b_r & b_b & b_g & \mu^+ & -(N_\mu)^C & 0
\end{pmatrix}_L \quad , \quad (14)
$$

$$
(\psi^{15})^{31}_L = \frac{1}{\sqrt{2}}
\begin{pmatrix}
0 & (t_g)^C & -(t_b)^C & -t_r & -b_r & -d_r \\
-(t_g)^C & 0 & (t_r)^C & -t_b & -b_b & -d_b \\
(t_b)^C & -(t_r)^C & 0 & -t_g & -b_g & -d_g \\
t_r & t_b & t_g & 0 & e^+ & -\tau^+ \\
b_r & b_b & b_g & -e^+ & 0 & (N_\tau)^C \\
d_r & d_b & d_g & \tau^+ & -(N_\tau)^C & 0
\end{pmatrix}_L \quad .
$$

6 Summary and discussion

We have constructed a complete set of generators for the special unitary group $SU(6)$ which is able to unify three forces of our nature. The generators have been derived from the first principle of group theory by assuming that the symmetry breaking occurs through the intermediate stage with $SU(3)_\text{C} \otimes SU(3)_\text{H} \otimes U(1)_\text{C}$ symmetry.

Using these generators, we have found the extended Gell-Mann Okubo relation which could accomodate all combinations of quantum numbers contained in the model including the new hyper-isospin. The relation leads to a unique configuration of fermions in the $\{6\}-$ and $\{\overline{15}\}-$plets. In order to fill in the multiplets completely we have introduced a new neutral fermion for each generation, N_ℓ, belongs to the triplet in $SU(3)_\text{H}$ sub-group. This leads to a natural interpretation that N_ℓ's are the heavy Majorana neutrinos which play an important role to enable the seesaw mechanism.

Further study should be done to obtain a complete lagrangian representing the allowed and newly predicted interactions in the framework of this GUT model and its phenomenological consequences. More detail investigation should also be per-

formed to realize two stages of symmetry breakings through for instance the Higgs mechanism. These points will be discussed in detail in the subsequent paper [9].

Acknowledgement

Appendix

Here, we provide a complete set of matrices which forms generators for $SU(6)$ group. The last λ_{35} is written in Eq. (4).

$$\lambda_1 = \left(\begin{array}{c|c} \begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{array} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (0)_{3\times3} \end{array} \right) \qquad \lambda_2 = \left(\begin{array}{c|c} \begin{array}{ccc} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{array} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (0)_{3\times3} \end{array} \right)$$

$$\lambda_3 = \left(\begin{array}{c|c} \begin{array}{ccc} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{array} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (0)_{3\times3} \end{array} \right) \qquad \lambda_4 = \left(\begin{array}{c|c} \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{array} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (0)_{3\times3} \end{array} \right)$$

$$\lambda_5 = \left(\begin{array}{c|c} \begin{array}{ccc} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{array} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (0)_{3\times3} \end{array} \right) \qquad \lambda_6 = \left(\begin{array}{c|c} \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{array} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (0)_{3\times3} \end{array} \right)$$

$$\lambda_7 = \left(\begin{array}{c|c} \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{array} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (0)_{3\times3} \end{array} \right) \qquad \lambda_8 = \frac{1}{\sqrt{3}} \left(\begin{array}{c|c} \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{array} & (0)_{3\times3} \\ \hline (0)_{3\times3} & (0)_{3\times3} \end{array} \right)$$

$$\lambda_9 = \left(\begin{array}{c|c} (0)_{3\times3} & \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \\ \hline \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} & (0)_{3\times3} \end{array} \right) \qquad \lambda_{10} = \left(\begin{array}{c|c} (0)_{3\times3} & \begin{array}{ccc} -i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \\ \hline \begin{array}{ccc} i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} & (0)_{3\times3} \end{array} \right)$$

$$\lambda_{11} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 1 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix} \qquad \lambda_{12} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & -i & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & i & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix}$$

$$\lambda_{13} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix} \qquad \lambda_{14} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & -i & 0 & 0 \\ \hline 0 & 0 & i & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix}$$

$$\lambda_{15} = \begin{pmatrix} & & & 0 & 1 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 1 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix} \qquad \lambda_{16} = \begin{pmatrix} & & & 0 & -i & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & i & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix}$$

$$\lambda_{17} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 1 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 1 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix} \qquad \lambda_{18} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & -i & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & i & 0 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix}$$

$$\lambda_{19} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & 1 & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix} \qquad \lambda_{20} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & -i & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & i & & (0)_{3\times3} & \\ 0 & 0 & 0 & & & \end{pmatrix}$$

$$\lambda_{21} = \begin{pmatrix} & & & 0 & 0 & 1 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 1 & 0 & 0 & & & \end{pmatrix} \qquad \lambda_{22} = \begin{pmatrix} & & & 0 & 0 & -i \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ i & 0 & 0 & & & \end{pmatrix}$$

$$\lambda_{23} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 1 \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 1 & 0 & & & \end{pmatrix} \qquad \lambda_{24} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & -i \\ & & & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & i & 0 & & & \end{pmatrix}$$

$$\lambda_{25} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & 1 \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & 1 & & & \end{pmatrix} \qquad \lambda_{26} = \begin{pmatrix} & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 0 & 0 & -i \\ \hline 0 & 0 & 0 & & & \\ 0 & 0 & 0 & & (0)_{3\times3} & \\ 0 & 0 & i & & & \end{pmatrix}$$

$$\lambda_{27} = \begin{pmatrix} & (0)_{3\times3} & & & (0)_{3\times3} & \\ \hline & & & 0 & 1 & 0 \\ & (0)_{3\times3} & & 1 & 0 & 0 \\ & & & 0 & 0 & 0 \end{pmatrix} \qquad \lambda_{28} = \begin{pmatrix} & (0)_{3\times3} & & & (0)_{3\times3} & \\ \hline & & & 0 & -i & 0 \\ & (0)_{3\times3} & & i & 0 & 0 \\ & & & 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{29} = \begin{pmatrix} & (0)_{3\times3} & & & (0)_{3\times3} & \\ \hline & & & 1 & 0 & 0 \\ & (0)_{3\times3} & & 0 & -1 & 0 \\ & & & 0 & 0 & 0 \end{pmatrix} \qquad \lambda_{30} = \begin{pmatrix} & (0)_{3\times3} & & & (0)_{3\times3} & \\ \hline & & & 0 & 0 & 1 \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_{31} = \begin{pmatrix} & (0)_{3\times3} & & & (0)_{3\times3} & \\ \hline & & & 0 & 0 & -i \\ & (0)_{3\times3} & & 0 & 0 & 0 \\ & & & i & 0 & 0 \end{pmatrix} \qquad \lambda_{32} = \begin{pmatrix} & (0)_{3\times3} & & & (0)_{3\times3} & \\ \hline & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & 1 \\ & & & 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_{33} = \begin{pmatrix} & (0)_{3\times3} & & & (0)_{3\times3} & \\ \hline & & & 0 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 0 & -i \\ & & & 0 & i & 0 \end{pmatrix} \qquad \lambda_{34} = \frac{1}{\sqrt{3}} \begin{pmatrix} & (0)_{3\times3} & & & (0)_{3\times3} & \\ \hline & & & 1 & 0 & 0 \\ & (0)_{3\times3} & & 0 & 1 & 0 \\ & & & 0 & 0 & -2 \end{pmatrix}$$

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